NEUTRINO, COSMOS, AND NEW PHYSICS

A.D. DOLGOV

INFN, Ferrara 40100, Italy
ITEP, 117218, Moscow, Russia

E-mail: dolgov@fe.infn.it

ABSTRACT

Observational manifestations of possible breaking of spin-statistics relation for neutrinos are considered. It is argued that bosonic neutrinos may form cosmological cold dark matter, improve agreement of BBN predictions with observations, make operative Z-burst model of ultra-high energy cosmic rays, etc. Restrictions for an admixture of bosonic component to neutrino which follow from double beta decay are discussed.

1. Introduction

There is an impressive symbiosis of the "weakest" and lightest of the known massive particles, neutrino, and Cosmos. Cosmology and astrophysics allow to study neutrino properties with an accuracy which, in many cases, is unaccessible in direct terrestrial experiments and, vice versa, neutrino helps to resolve some cosmological and astrophysical mysteries[1,2].

Cosmology allows to put a very stringent upper bound on neutrino mass at the level of about 1 eV, for a review see e.g. papers[3]. The bound on the amplitude of possible right handed currents and the mass of right intermediate bosons found from the analysis of Big Bang Nucleosynthesis (BBN) is orders of magnitude better than those obtained in laboratories. The same is true for neutrino magnetic moments and mixing of the usual active neutrinos with hypothetical sterile ones which are restricted by BBN[1] and stellar evolution[4].

On the other hand, neutrino contributes to cosmology providing hot dark matter, but not the necessary cold one, if physics is normal. Neutrino may be related to dark energy[5], and be responsible, at least partly, for ultra-high energy cosmic rays beyond GZK cut-off through the Z-burst mechanism[6]. The large mixing angle solution to the solar neutrino anomaly excludes noticeable lepton asymmetry of the universe[7].

These lists are far from being complete but this is not the main subject of this talk. Instead of these rather well known topics I would like to talk about new, though quite speculative issues, related to effects of possible breaking of neutrino statistics in cosmology. The content of the talk is strongly based on ref.[8].

The first question is why neutrino? First of all, neutrino is the only known particle indicating to new physics. As is known from observation of neutrino flavor oscillations leptonic flavor charges, electronic, muonic, and tauonic, are not conserved. Neutrino is the only observed particle for which Majorana mass is possible and, as a result

total leptonic charge could be non-conserved. There are experimental indications to a possible leptonic charge non-conservation from neutrinoless double beta decay[9)]. It may mean that neutrino communicates with a hidden sector of the particle world and is a messenger of new physics from the hidden sector. It could be that no sacred principles are respected in the hidden world and neutrinos brings us exotic possibilities of breaking CPT theorem[10)] or Lorentz invariance, which are actively discussed in the recent years. Since at the present days cosmology is quickly becoming precise science, maybe cosmos will bring through neutrinos new surprising physics.

The most exciting possibility which, in particular, may lead to violation of CPT and Lorentz invariance and even to much more drastic consequences is a breaking of the spin-statistics relation for neutrinos. Ironically the particle brought to this world by Pauli may violate Pauli exclusion principle. In fact Pauli and Fermi repeatedly asked the question if spin-statistics relation could be not exact and electrons were a little bit different.

Possible violation of exclusion principle for the usual matter, i.e. for electrons and nucleons was discussed in a number of papers at the end of the 80th[11)]. Efforts to find a more general than pure Fermi-Dirac or Bose-Einstein statistics[12)] were taken but no satisfactory theoretical frameworks had been found. Experimental searches of the Pauli principle violation for electrons [13)] and nucleons [14)] have also given negative results.

If one assumes that spin-statistics relation is broken while otherwise remaining in the frameworks of the traditional quantum field theory then immediately several deep theoretical problems would emerge:
1) non-locality;
2) faster-than-light signals;
3) non-positive energy density and possibly unstable vacuum;
4) maybe breaking of unitarity;
5) broken CPT and Lorentz invariance (as mentioned above).

Either these consequences (if they indeed were realized) exclude any violation of spin-statistics theorem and discussion of this violation should be forbidden or they open an exciting space for further research and development. An answer to that is first of all a matter of experiment which may either exclude or confirm the drastic assumption of breaking the spin-statistics relation. As for observational manifestations of the mentioned phenomena they should be weak because they are induced by weakly interacting neutrinos and, moreover, in higher orders of perturbation theory.

Perturbative expansion of the scattering matrix has the well known form:

$$S = 1 + \sum_n \frac{(-i)^n}{n!} \int \Pi d^4 x_j T \left\{ \mathcal{H}(x_1)...\mathcal{H}(x_n) \right\} \tag{1}$$

where $T\{...\}$ means time-ordered product of operators inside brackets. Lorentz invariance is ensured if Hamiltonian, $\mathcal{H}$, is bosonic operator, i.e. it commutes with itself

if separated by the light cone, see e.g.[15]. However, for bosonic or partly bosonic neutrinos the effective Hamiltonian responsible even for the simple reaction $e+p \leftrightarrow n+\nu$ is not bosonic and observables do not commute for space-like separation and locality breaks. Presumably unitarity is maintained because Hamiltonian remains Hermitian.

Another possibility that Hamiltonian/Lagrangian approach and least action principle are applicable only approximately and theory is drastically modified, while the observable effects may still be small.

So let us postpone discussion of (non-existing) theory and consider phenomenology of neutrinos obeying Bose or mixed statistics[8]. What can we buy for this price?

2. Dark matter .

It is well known that the usual fermionic neutrinos cannot form cosmological cold dark matter for any spectrum of primordial fluctuations and arbitrary self-interaction. This conclusion is based on the Tremain-Gunn bound[16] which does not allow to fill galaxies with sufficiently many light fermions (satisfying the Gerstein-Zeldovich bound[17]) to account for the observed hidden mass. Thus we face the following dark matter dilemma:
1) new particles and old (normal) physics
2) old particles (neutrinos) and very new physics.

To make the cosmological cold dark matter neutrinos must form Bose condensate in the early universe. To this end a very large lepton asymmetry is necessary with

$$\frac{|n_\nu - n_{\bar{\nu}}|}{n_\gamma} \sim 100 \tag{2}$$

Such asymmetry might be created in a version of the Affleck and Dine[18] scenario.

A large asymmetry allows to fill up the present day universe by a huge number of cosmic neutrinos such that they would be able to make all CDM, $\Omega_{CDM} \approx 0.25$, if

$$n_\nu \sim 10^4 \text{ cm}^{-3} \tag{3}$$

The spectrum of cosmic background neutrinos, if they are bosonic, would be very much different from the fermionic ones because the Bose equilibrium distribution has the form:

$$f_{\nu_b} = \frac{1}{\exp[(E - \mu_\nu)/T - 1]} + C\delta(k), \tag{4}$$

where $\mu = m_\nu$ is the maximum value of chemical potential of bosonic neutrinos equal to their mass. The condensate amplitude C does not depend upon neutrino energy but may depend upon time. One sees that the bulk of the condensed neutrinos is cold. In galaxies the neutrino number density would be about 6 orders of magnitude larger than the average cosmological number density, i.e.

$$n_\nu^{(gal)} \sim 10^{10} \, (m_\nu/0.1 \, \text{eV}) \, \text{cm}^{-3}. \tag{5}$$

Structure formation with Bose condensed light bosons with the usual integer spin was considered in ref.[19]. The model well reproduces the essential features of the observed large scale structure. Since the picture is spin independent the same must be true as well for bosonic neutrinos.

The results and numerical estimates presented in this section are true for purely bosonic neutrinos however, as we see below, experiments on two neutrino double beta decay seem to exclude 100% breaking of statistics and at least some fermionic fraction must be present in a neutrino. It makes the model noticeably more cumbersome, but less vulnerable.

3. Equilibrium distribution for mixed statistics

The statistics dependent term in kinetic equation for the reaction $1 + 2 \leftrightarrow 3 + 4$ has the form

$$F = f_1(p_1)f_2(p_2)[1 \pm f_3(p_3)][1 \pm f_4(p_4)] - f_3(p_3)f_4(p_4)[1 \pm f_1(p_1)][1 \pm f_2(p_2)] \quad (6)$$

where f_j is the distribution function of particle j. This expression is valid in the case of T-invariant theory when the detailed balance condition is fulfilled. Since T-invariance is broken, kinetic equation is modified but the equilibrium distributions $f_j^{(eq)}$ remain the same canonical Bose and Fermi ones as in T-invariant theory [20]. This statement is based on the unitarity of S-matrix. If the spin-statistics relation is broken, as a result the unitarity may also be broken. If this is the case, then a breaking of T-invariance may create large deviations from the standard equilibrium distribution functions of neutrinos.

In what follows we neglect complications related to a violation of T-invariance. In the case that neutrino obeys pure Bose statistics its equilibrium distribution is given by the standard Bose form (4). Indeed, it is easy to see that for this distribution F vanishes and together with it the collision integral vanishes too. However, the form of equilibrium distribution for mixed statistics is not so evident. It depends upon an assumption about F for particles obeying mixed statistics. We do not have rigorous arguments in favor of one or other form for F and as a reasonable guess assume that the factor depending upon the neutrino statistics in F changes as

$$(1 - f_\nu) \to c^2(1 - f_\nu) + s^2(1 + f_\nu) \quad (7)$$

where $c = \cos\gamma$, $s = \sin\gamma$ and γ is some mixing angle characterizing admixture of wrong statistics.

Another possibility for description of mixed statistics in kinetic equation could be

$$(1 - f_\nu) \to c^2(1 - c^2 f_\nu) + s^2(1 + c^2 f_\nu). \quad (8)$$

However, it is easy to see that these two seemingly reasonable possibilities (7) and (8) are identically equivalent. In both cases

$$(1 - f_\nu) \to (1 - \kappa f_\nu) \quad (9)$$

where

$$\kappa = c^2 - s^2 = \cos 2\gamma \qquad (10)$$

We call κ the Fermi-Bose mixing parameter[21]. One can check that in the case of mixed statistics introduced to kinetic equation according to (9) the equilibrium distribution takes the form[21]:

$$f_\nu^{(eq)} = [\exp(E/T) + \kappa]^{-1} . \qquad (11)$$

where κ runs from $+1$ to -1 corresponding respectively to Fermi and Bose limits. The intermediate value $\kappa = 0$ corresponds to Boltzmann statistics.

If $-1 < \kappa < 0$, the maximum value of the chemical potential may be bigger than the neutrino mass:

$$\mu^{(max)} = m_\nu - T \ln(-\kappa) \qquad (12)$$

Bose condensation might take place for negative κ only.

Another possible form of a modification of statistics dependent factor in kinetic equation would emerge if we assume that there are two neutrino fields with the same mass and different statistics, fermionic and bosonic. The Lagrangian would always depend upon two independent field operators in the combination:

$$\psi_\nu = c\psi_b + s\psi_f, \qquad (13)$$

where $\psi_{f,b}$ are respectively bosonic and fermionic operators. In this case kinetic equation would contain two different terms:

$$c^2 f_f (1 - f_f) \qquad (14)$$

and

$$s^2 f_b (1 - f_b) \qquad (15)$$

Equilibrium distributions would be canonical ones, e.g. for vanishing chemical potential they are:

$$f_{f,b} = 1/\left[\exp\left(E/T\right) \pm 1\right] \qquad (16)$$

but the number of states in equilibrium becomes doubled. On the other hand, the probability of e.g. neutron beta-decay remains the same as in the standard theory because $c^2 + s^2 = 1$.

4. Big Bang Nucleosynthesis

The impact of purely bosonic neutrinos on BBN was considered earlier in paper[22]. The effects of mixed statistics described by the equilibrium distribution (11) were calculated in our work[21]. The equilibrium energy density of bosonic neutrinos at $T \gg m_\nu$ is 8/7 of the energy density of fermionic neutrinos and thus the change of

statistics would lead to an increase of the effective number of neutrino species at BBN by $\Delta N_\nu = 3/7$ (for three neutrinos). On the other hand, a larger magnitude of the neutrino distribution function and the fact that it enters kinetic equation (see (6)) as $(1+f_\nu)$ instead of $(1-f_\nu)$ makes the weak reactions of neutron-proton transformations faster and hence the n/p freezing temperature becomes lower. This effect dominates and as a result the effective number of massless species becomes smaller than 3. According to the calculations of ref.[21] the effective number of neutrino species in the case of pure Bose statistics becomes $N_{eff} = 2.43$, practically independently on the value of the baryon-to-photon ratio $\eta = n_B/n_\gamma$.

The effective number of neutrino species determined by the comparison of the calculated abundance of primordial ^4He with the standard result is presented in the upper panel of fig. 1 as a function of κ. However, the effect of change of statistics cannot be described by a simple change in N_ν if other light elements are included. In the lower panel of fig. 1 the relative changes of the abundances of ^2H, ^4He, and ^7Li are presented. As expected the mass fraction of ^4He drops down, while the amount of ^2H goes up. A higher deuterium abundance can be explained by a slower conversion of deuterium to heavier elements due to fewer neutrons and faster cosmological expansion at $T \approx 0.8 * 10^9$ when the light elements have been formed.

At $\kappa = -1$ we find for ^{4}He: $Y_p = 0.240$, which makes much better agreement with the value extracted from observations (for a review of the latter see *e.g.* [23]). Different helium observations yield different results, *e.g.*, ref. [24] finds $Y = 0.238 \pm 0.002$, and ref. [25] finds $Y = 0.2421 \pm 0.0021$ (1σ, only statistical error-bars). These results are shown in figure 2 as the skew hatched (yellow) region. Whether the existing helium observations are accurate or slightly systematically shifted will be tested with future CMB observations [26].

The amount of ^{2}H rises at most to $X_{^2H}/X_H = 2.5 \cdot 10^{-5}$ and the agreement between BBN and WMAP data remains good, bearing in mind the observational uncertainties. Primordial ^{7}Li drops down to $X_{^7Li}/X_H = 4.55 \cdot 10^{-10}$, again slightly diminishing the disagreement between theory and observations.

We see that at the present time BBN does not exclude even a pure bosonic nature of all three neutrinos. Furthermore, the agreement between the value of the baryonic mass density, η, inferred from CMBR and the predicted abundances of ^{4}He, ^{2}H, and ^{7}Li becomes even better. In other words, in the standard BBN model there is an indication of disagreement between observations of ^{4}He and ^{2}H - they correspond to different values of η with the observed abundances of ^{4}He indicating a smaller value than given by CMBR, while ^{2}H agrees with CMBR. Motivated by these results the value of $\Delta N_\nu = -0.7 \pm 0.35$ was suggested in ref. [27]. In the case of predominantly bosonic neutrinos, as discussed above, the discrepancy between ^{2}H, ^{4}He, and CMBR disappears.

The results presented in this section are obtained for negligible chemical potential of electronic neutrinos. On the other hand, formation of cosmological neutrino
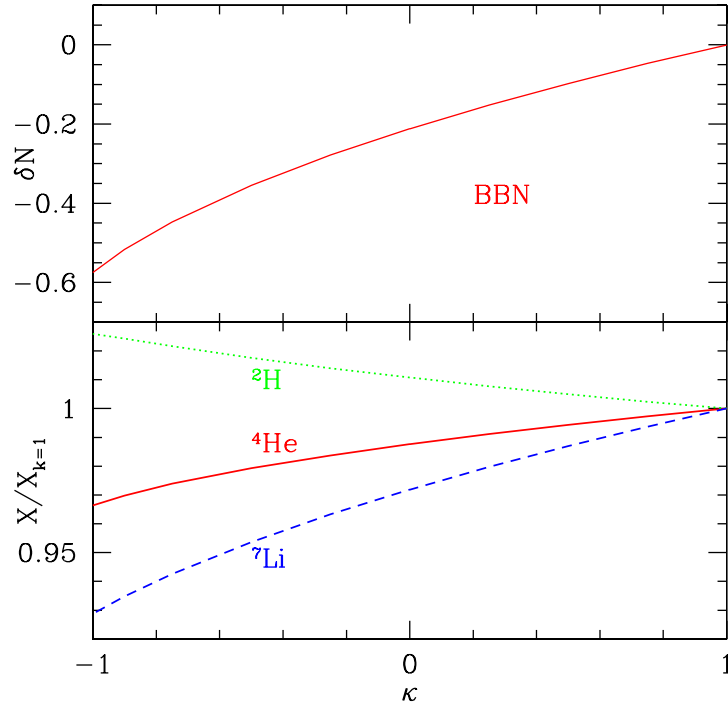
Figure 1: Upper panel: the change in the effective number of degrees of freedom which corresponds to the change of the ^{4}He abundance as a function of the effective Fermi-Bose parameter κ. Lower panel: the relative change of the primordial abundances of deuterium, helium-4, and lithium-7, as functions of κ. We take $\eta = n_B/n_\gamma = 6.5 \cdot 10^{-10}$.
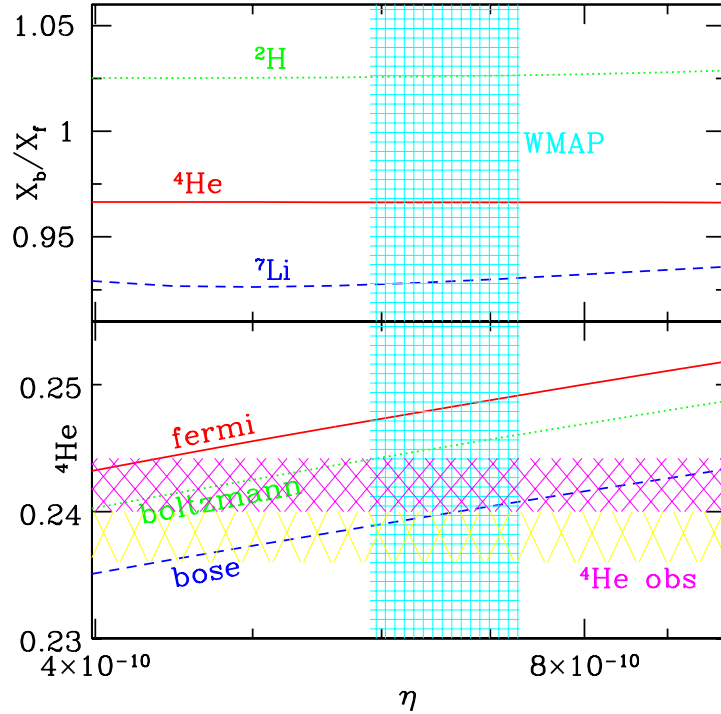
Figure 2: Upper panel: the ratios of abundances of different elements in the cases of purely bosonic neutrinos with respect to the standard fermionic case as functions of the baryon number density, η. The vertically hatched (cyan) region shows the WMAP 2σ determination of η. Lower panel: the absolute abundance of ^{4}He as a function of η for the purely bosonic, Boltzmann, and fermionic neutrino distributions, corresponding to $\kappa = -1, 0, +1$ respectively. The two skew hatched regions show the observation of primordial helium from ref.[24] (lower, yellow) and ref.[25] (upper, magenta), which marginally overlap at 1σ.

condensate discussed in sec. 2 demands the maximum value of μ_ν given by eq. (12). As is known[7], BBN allows at most $\mu/T = 0.07$ for any neutrino flavor. This implies $\kappa > 0.9$. Such a large admixture of a wrong bosonic state to ν_e is most probably excluded by the data on double beta decay (see below, sec. 7). However, one may still hope to save the neutrino cold dark matter (νCDM) if the mixing angle determined from the decay is different from that that enters neutrino kinetics (see discussion in sec. 8. Another possibility to save νCDM is to assume that the chemical potentials of ν_μ or μ_τ are much larger than that of ν_e. The latter are very weakly restricted by BBN and only the large mixing between neutrino flavors equalizes all chemical potentials. However, the change of neutrino statistics may lead to a different refraction index in the primeval plasma and to suppression of the transformation of $\nu_{\mu,\tau}$ into ν_e. There is also a more conventional way to suppress neutrino flavor oscillations in the primeval plasma introducing neutrino coupling to light pseudo-goldstone boson, Majoron. The effective potential of neutrinos induced by the Majoron exchange would suppress flavor transformations in the cosmological plasma[28]. This would allow to have large chemical potentials of $\nu_{\mu,\tau}$ and small chemical potential of ν_e.

5. Astrophysical consequences

Neutrino statistics plays key role in the environments where neutrinos form dense degenerate gases. Direct test of the "bosonic" nature of neutrinos can be provided by precise measurements of the neutrino energy spectrum from supernova. Generically, the spectrum of bosonic neutrinos should be more narrow. To establish the difference one needs to measure the spectrum both in the low, $E < 3T$, and in the high, $E > 3T$ energy parts.

A violation of the exclusion principle can influence dynamics of the SN collapse. According to the usual scenario at the initial stages (formation of the hot proto-neutron star) the neutronization leads to production of high concentration of the electron neutrinos which are trapped in the core. The chemical potential of these neutrinos (due to the Pauli principle) can reach 70 - 100 MeV. These neutrinos heat the medium and diffuse from the core. Violation of the Pauli principle allows for the neutronization neutrinos to be produced with lower energies. These neutrinos escape easier the star leading to faster cooling and lower central temperatures. The evolution of the lepton number would change as well.

High neutrino density in the condensate (especially if an additional clustering occurs) enhances the rate of the Z^0-bursts produced by the annihilation of the ultra high energy (UHE) cosmic neutrinos on the relic neutrinos [6]. This in turn, enhances production of the UHE cosmic rays, and may help to explain the cosmic ray evens above the GZK cut-off.

Charge asymmetric neutrino condensate may produce a strong refraction of the high energy neutrinos from remote sources (active galactic nuclei, gamma ray bursters).

Apart from lensing, one may expect a substantial impact on neutrino oscillations [29].

Since the density of dark matter in galaxies is about 6 orders of magnitude larger than their average cosmological energy density, a condensation of cold neutrinos around the Earth might have an effect on the end point of the beta decay spectra, in particular, in the tritium decay experiments on search for neutrino mass [30].

6. Double beta decay

In contrast to electrons and nucleons which form atoms and nuclei, where the effects of statistics are of primary importance, it is difficult to observe processes with identical neutrinos. A realistic reaction for the test of neutrino statistics can be the two-neutrino double beta decay,

$$A \rightarrow A' + 2\bar{\nu} + 2e^- \tag{17}$$

(or similar with production of antineutrinos and positrons). The probability of the decay as well as the energy spectrum and angular distribution of electrons should be affected by the change of neutrino statistics.

To have a formalism for description of identical neutrinos one needs to specify operators of neutrino creation/annihilation. We assume that they consist of two parts, fermionic, $\hat{f}$, and bosonic $\hat{b}$ for operators of annihilation, $\hat{a} = \hat{f} + \hat{b}$. Its Hermitian conjugate could naturally be the operator of neutrino creation. Correspondingly we define one neutrino state as:

$$|\nu\rangle = \hat{a}^+|0\rangle \equiv c_1\hat{f}^+|0\rangle + s_1\hat{b}^+|0\rangle = c|f\rangle + s|b\rangle \tag{18}$$

where $|f\rangle$ and $|b\rangle$ are respectively one particle fermionic and bosonic states and $c_1 = \cos\delta$ and $s_1 = \sin\delta$. It would be natural to expect that δ is equal to γ introduced above in eq. (7) but we cannot prove it formally.

To describe the two-neutrino state one needs to specify the relevant commutators which, we postulate, have the following form:

$$\hat{f}\hat{b} = e^{i\phi}\hat{b}\hat{f}, \quad \hat{f}^+\hat{b}^+ = e^{i\phi}\hat{b}^+\hat{f}^+, \quad \hat{f}\hat{b}^+ = e^{-i\phi}\hat{b}^+\hat{f}, \quad \hat{f}^+\hat{b} = e^{-i\phi}\hat{b}\hat{f}^+, \tag{19}$$

where ϕ is an arbitrary phase. The two neutrino state is natural to define as

$$|k_1, k_2\rangle = \hat{a}_1^+\hat{a}_2^+|0\rangle \tag{20}$$

The matrix element of the decay of nucleus A into $2\nu + 2e + A'$ may be possibly taken in the usual way:

$$A_{2\beta} = \langle k_1, k_2, 2e, A' \Big| \int d^4x_1 d^4x_2 \psi_\nu(x_1)\psi_2(x_2)\mathcal{M}(x_1, x_2)\Big|0, A\rangle. \tag{21}$$

After making the necessary commutating according to eq. (19) we obtain:

$$A_{2\beta} = A_- \left[c_1^4 + c_1^2 s_1^2 \left(1 - \cos\phi\right)\right] + A_+ \left[s_1^4 + c_1^2 s_1^2 \left(1 + \cos\phi\right)\right]. \tag{22}$$

where A_- and A_+ are respectively antisymmetric (fermionic) and symmetric (bosonic) parts of two neutrino emission. It is easy to see that the amplitude can be parametrized as

$$A_{2\beta} = \cos^2 \chi \, A_- + \sin^2 \chi \, A_+, \tag{23}$$

where $\cos^2 \chi = c_1^4 + c_1^2 s_1^2 (1 - \cos \phi)$ and $\sin^2 \chi = s_1^4 + c_1^2 s_1^2 (1 + \cos \phi)$. The probability of the double beta decay integrated over neutrino momenta evidently does not contain interference between A_+ and A_- and is equal to:

$$W_{tot} = \cos^4 \chi \, W_- + \sin^4 \chi \, W_+, \tag{24}$$

where $W_\pm$ are proportional to $|A_\pm|^2$.

The probability of decay into unusual bosonic neutrinos is proportional to the bi-linear combinations of the type $K_m K_n$, $K_m L_n$, $L_m L_n$, where

$$K_m^b \equiv [E_m - E_i + E_{e1} + E_{\nu1}]^{-1} - [E_m - E_i + E_{e2} + E_{\nu2}]^{-1},$$
$$L_m^b \equiv [E_m - E_i + E_{e2} + E_{\nu1}]^{-1} - [E_m - E_i + E_{e1} + E_{\nu2}]^{-1}. \tag{25}$$

Here the upper index b indicates that the results are applicable to bosonic neutrinos, E_i is the energy of the initial nuclei, E_m is the energy of the intermediate nucleus state m, E_{ei}, and $E_{\nu i}$ are the energies of electrons and neutrinos respectively. The minus signs between the two terms in the above expressions are due to the bosonic character of neutrinos; in the case of fermionic neutrinos we would have plus signs [31]. For electrons we assume the normal Fermi statistics.

In the case of $0^+ \to 0^+$ transitions the combinations K_m and L_m can be approximated by

$$K_m^b \approx \frac{E_{e2} - E_{e1} + E_{\nu2} - E_{\nu1}}{(E_m - E_i + E_0/2)^2}, \qquad L_m^b \approx \frac{E_{e1} - E_{e2} + E_{\nu2} - E_{\nu1}}{(E_m - E_i + E_0/2)^2}, \tag{26}$$

whereas for the fermionic neutrinos

$$K_m^f \approx L_m^f \approx \frac{2}{E_m - E_i + E_0/2}. \tag{27}$$

Here $E_0/2 = E_e + E_\nu$ is the average energy of the leptonic pair. Appearance of the differences of the electron and neutrino energies in eq. (26) leads to a suppression of the total probability. It also modifies the energy distributions of electrons. The probabilities of the transitions $0^+ \to 2^+$ are proportional to the combinations $(K_m - L_m)(K_n - L_n)$, where

$$(K_m^b - L_m^b) \approx \frac{2(E_{e2} - E_{e1})}{(E_m - E_i + E_0/2)^2}. \tag{28}$$

In the case of fermionic neutrinos the combination has an additional factor $(E_{\nu2} - E_{\nu1})/(E_m - E_i + E_0/2)$ and the suppression is stronger.

A simple estimate shows that the probability of $0^+ \to 0^+$-transition for bosonic neutrinos is suppressed by $1/250$ for ^{56}Ge and by $1/10$ for ^{100}Mo. Theoretically the total decay rate is known with the accuracy within a factor of few. This probably allows to exclude a 100% bosonic neutrino. However, the fraction of bosonic neutrino can still be very high. According to our preliminary calculations[8,32] the value of the mixing angle can be as large as:

$$\sin^2 \chi \leq 0.7 \tag{29}$$

For $0^+ \to 2^+$ the situation is opposite: bosonic neutrinos are more efficiently produced. However, no interesting bound is obtained in this case because the statistics for these decays is much lower.

One can use the data on the spectrum of the emitted electrons, either single electron spectrum or distribution over the total energy of both electrons. The spectra do not have any noticeable ambiguity related to unknown nuclear matrix elements and the present day accuracy is at the level of 10%. Potentially their analysis may improve the above quoted limit (29) or indicate the existence of a "bosonic" admixture to neutrinos. Some already observed anomalies may be interpreted as hints supporting the latter.

Unfortunately we cannot say at the present stage how the Fermi-Bose parameter introduced above (10) is related to the mixing angle χ. Even if we assume that the mixing angle in neutrino kinetics (7) is the same as in the definition of neutrino states (18), the unknown value of the angle ϕ which enters the commutation relations (19) and upon which depends the angle χ (23) makes the relation between κ and χ rather arbitrary.

7. Theoretical problems and discussion

Mentioned above ambiguities are related to intrinsic problems of formulation a theory with mixed statistics. Working at a naive level, as we did above, it is even difficult to define the properly normalized particle number operator. According to eqs. (18) and (20) it is natural to define the n identical neutrino state as

$$|n\rangle = \left(c_1 f^+ + s_1 b^+\right)^n |0\rangle \tag{30}$$

The normalization of this state is

$$\langle n|n\rangle = s_1^{2(n-1)} \left[n! s_1^2 + (n-1)! c_1^2 \left(\frac{\sin\left(n\phi/2\right)}{\sin\left(\phi/2\right)} \right)^2 \right] \tag{31}$$

If we introduce the particle number operator in the usual way:

$$\hat{n} = a^+ a, \tag{32}$$

then its diagonal matrix elements would be

$$\langle n|\hat{n}|n\rangle = s_1^{2(n-1)} \left[n\, n!s_1^4 + 2n!c_1^2 s_1^2 \cos\frac{\phi(n-1)}{2}\frac{\sin n\phi/2}{\sin\phi/2} + \right.$$

$$\left. c_1^2\left(n!s_1^2 + (n-1)!(c_1^2 - s_1^2)\right)\left(\frac{\sin(n\phi/2)}{\sin(\phi/2)}\right)^2\right] \tag{33}$$

The particle number operator, as introduced above, has reasonable and self-consistent interpretation only for the case of pure statistics, while for mixed statistics it even does not commute with Hamiltonian if the latter, or operator $\hat{n}$, or both are not somehow modified.

There are no problems with reactions where only one neutrino is involved, but serious difficulties may arise with two neutrino reactions, as e.g. $\nu l \to \nu l$ or $\bar{\nu}\nu \to \bar{l}l$, even if the participating neutrinos are not in an identical state. The amplitude of νe-elastic scattering in the usual approach is given by the expansion of the T-exponent of the action and is described by two diagrams differing by an interchange of emission and absorption points. If taken literally, the diagrams with $W^{\pm}$-exchange would give vanishing amplitude for purely bosonic neutrinos [a]. In this case only Z-exchange would contribute to $\nu_e e$-scattering and the cross-sections of $\nu_\mu e$ and $\nu_e e$-scattering would be equal. Reactor neutrino experiments are consistent with the standard value of $\nu_e e$ cross-section and seem to exclude the possibility of purely bosonic ν_e. Using these data one can put a rather strong bound on bosonic admixture to electronic neutrino. On the other hand, perturbation theory with non-bosonic Hamiltonian may need to be modified and the above conclusion of vanishing of the amplitude of scattering of pure bosonic neutrinos on electrons would be invalid.

It is unclear if all these problems can be resolved in a simple way or drastic modifications of the underlying theory is necessary, which is a nontrivial task because the observed consequences of the theory must not be destroyed.

The presentation in the previous sections and in the original paper[8] was on pure (and poor) phenomenological level. For example if neutrinos have mixed statistics then in double beta decay the symmetry of the final state of neutrinos is mixed: symmetric with wight a_+ and antisymmetric with the weight a_-. It seems plausible that these weights are respectively $\cos^4\chi$ and $\sin^4\chi$ as argued in the previous section, eq. (24), simply on the basis on the normalization arguments.

Similar reasoning is possible for kinetic effects, eqs. (7,8). There are no rigorous theoretical arguments in favor of such description but the result (11) for the equilibrium distribution in the case of mixed statistics looks quite beautiful. Moreover, the fact that two "reasonable" (or natural) ways of description (7) and (8) give the same

---

[a]This was noticed also by F. Vissani at this Conference.

result is an argument in favor of their validity.

8. Conclusion

There is no consistent theoretical frameworks for description of mixed neutrino statistics and even in the case of purely bosonic neutrinos the fermionic property of the Hamiltonian would make a possible future theory quite unusual if it will ever be formulated. Still independently on theory there could be some predictions testable by experiment. So to summarize we will conclude that:

1. The suggestion of bosonic or mixed statistics for particles (neutrinos) with half integer spin looks exciting but opens a Pandora box of serious theoretical problems, which may be impossible to resolve without revolutionary modification of the standard theory. Such modification looks especially difficult in the case of mixed statistics.

2. The suggested mixture of statistics allows to break plenty of sacred principles, as e.g. Lorentz invariance, CPT-theorem, locality, etc, which are actively discussed now.

3. Bosonic neutrinos open a possibility of making all cosmological dark matter out of neutrinos in accordance with Occam's razor: "Plurality should not be posited without necessity."

4. "Bosonization" of neutrinos leads to effective number of neutrino species at BBN smaller than 3 and makes an agreement of the BBN calculations with the data noticeably better.

5. Analysis of accumulated and accumulating data on two neutrino double beta decay could restrict the admixture of wrong statistics to neutrinos or to indicate a violation of spin-statistics relation.

6. Last, but not the least, if the validity of spin-statistics theorem has been studied for the usual matter, electrons and nucleons it surely worth studying for neutrinos. The possibility that statistics is modified for neutrinos seems more plausible because neutrino is a natural particle to be a messenger from hidden sector of physics where some principles respected in our world can be violated.

9. References

1) A.D. Dolgov, *Phys. Rept.* **370** (2002) 333, hep-ph/0202122;
   A.D. Dolgov, *Surveys High Energ. Phys.* **17** (2002) 91, hep-ph/0208222.

2) G.G. Raffelt, *AIP Conf. Proc.* **721** (2004) 130;
   S. Hannestad, *New J. Phys.* **6** (2004) 108, hep-ph/0404239.

3) S. Hannestad, Talk given at NOW 2004: Neutrino Oscillation Workshop, Conca Specchiulla, Otranto, Italy, 11-17 Sep 2004, hep-ph/0412181;
   R.H. Brandenberger, A. Mazumdar, M. Yamaguchi, *Phys. Rev.* **D69** (2004) 081301, hep-ph/0401239.

4) G.G. Raffelt, *Stars as laboratories for fundamental physics: the astrophysics of neutrinos, axions, and other weakly interacting particles*, Chicago, USA: Univ. Press. (1996);
   G.G. Raffelt, astro-ph/0302589.

5) D.B. Kaplan, A.E. Nelson, N. Weiner, *Phys. Rev. Lett.* 93 (2004) 091801, hep-ph/0401099;
   see also a talk by A. Masiero at this Conference, *Varying neutrino mass and dark energy.*

6) D. Fargion, B. Mele, A. Salis, Astrophys. J. **517** (1999) 725, astro-ph/9710029;
   T. J. Weiler, *Phys. Rev. Lett.* **49** (1982) 234; Astropart. Phys. **11** (1999) 303, hep-ph/9710431;
   Z. Fodor, S.D. Katz, A. Ringwald, "Oulu 2002, Beyond the desert" p. 567, hep-ph/0210123;
   D.V. Semikoz, G. Sigl, *JCAP* **0404** (2004) 003, hep-ph/0309328.

7) A.D. Dolgov, S.H. Hansen, S. Pastor, S.T. Petcov, G.G. Raffelt, D.V. Semikoz, *Nucl. Phys.* **B632** (2002) 363, hep-ph/0201287.

8) A.D. Dolgov, A.Yu. Smirnov, hep-ph/0501066.

9) H.V. Klapdor-Kleingrothaus, et al, *Eur. Phys. J.* A**12** (2001) 147; H.V. Klapdor-Kleingrothaus, A. Dietz, I.V. Krivosheina, O. Chkvorets *Nucl. Instrum. Meth.* A**522** (2004) 371;
   The NEMO Collaboration (R. Arnold et al.) *JETP Lett.* **80** (2004) 429; [*JETP Lett.* **80** (2004) 377; hep-ex/0410021].

10) V.A. Kostelecky, M. Mewes, *Phys. Rev.* D**70** (2004) 031902, hep-ph/0308300, Phys. Rev. D**69** (2004) 016005 hep-ph/0309025;
    N.E. Mavromatos, hep-ph/0402005, Talk given at 2nd International Workshop on Neutrino Oscillations in Venice (NO-VE 2003), Venice, Italy, 3-5 Dec 2003. "Venice 2003, Neutrino oscillations" 405.

11) A.Yu. Ignatiev, V.A. Kuzmin, *Yad. Fiz.* **46** (1987) 786 [*Sov. J. Nucl. Phys.* **46** (1987) 786]; *JETP Lett.* **47** (1988) 4;

L.B. Okun, *Pis'ma ZhETF*, **46** (1987) 420 [*JETP Lett.* **46** (1987) 529]; *Yad. Fiz.* **47** (1988) 1192;

O.W. Greenberg, R.N. Mohapatra, *Phys. Rev. Lett.* **59** (1987) 2507, **62** (1989) 712, *Phys. Rev.* D**39** (1989) 2032;

A.B. Govorkov, *Theor. Math. Phys.* **54** (1983) 234;

A.B. Govorkov, *Phys. Lett.* A**137** (1989) 7;

L.B. Okun, *Uspekhi Fiz. Nauk*, **158** (1989) 293 [*Sov. Phys. Usp.* **32** (1989) 543], *Comments Nucl. Part. Phys.* **19** (1989) 99.

12) H.S. Green, *Phys. Rev.* **90** (1953) 270;

O.W. Greenberg, *Phys. Rev. Lett.* **13** (1964) 598;

O.W. Greenberg, A.M.L. Messiah, *Phys. Rev.* **138** (1965) B1155;

R.N. Mohapatra, *Phys. Lett* B**242** (1990) 407;

O.W. Greenberg, A.K. Mishra, *Phys. Rev.* D**70** (2004) 125013, math-ph/0406011.

13) M. Goldhaber, G. Scharff-Goldhaber, *Phys. Rev.* **73** (1948) 1472;

E. Fishbach, T. Kirsten and O. Shaeffer, *Phys. Rev. Lett.* **20** (1968) 1012;

F. Reines, H.W. Sobel, *Phys. Rev. Lett.* **32** (1974) 954;

B.A. Logan, A. Ljubicic, *Phys. Rev.* C**20** (1979) 1957;

R.D. Amado, H. Primakoff, *Phys. Rev.* C**22** (1980) 1338;

E. Ramberg, G. Snow, *Phys. Lett.* B**238** (1990) 438;

K. Deilamian, J. D. Gillaspy, D. Kelleher, *Phys. Rev. Lett.* **74** (1995) 4787;

V. M. Novikov, A. A. Pomansky, E. Nolte, *JETP Lett.* **51** (1990) 1;

V.M. Novikov, et al., *Phys. Lett.* B**240** (1990) 227.

14) R. Plaga, *Z. Phys.* A **333** (1990) 397; A.C. Barabash, V.N. Kornoukhov, Yu.M. Tsipenyuk, V.A. Chapyzhnikov, *Pis'ma ZhETF*, **68** (1998) 104 [*JETP Lett.* **68** (1998) 112];

The NEMO Collaboration (R. Arnold et al.) *Eur. Phys. J.* A**6** (1999) 361;

E. Baron, R.N. Mohapatra, V.L. Teplitz, *Phys. Rev.* D**59** (1999) 036003;

Borexino collaboration, H.O.Back, et al, hep-ph/0406252, *European Physical Journal* C **37** (2004) 421.

15) C. Itzykson, J.-B. Zuber, *Quantum Field Theory*, p. 150; McGraw-Hill, 1985; S. Weinberg, *The Quantum Theory of Fields*, pp. 144, 236; Cambridge University Press, 1995.

16) S. Tremaine, J.E. Gunn, *Phys. Rev. Lett.* **42** (1979) 407.

17) S.S. Gerstein, Ya.B. Zeldovich, *Pisma Zh. Eksp. Teor. Fiz.* **4** (1966) 174 [*JETP Lett.* **4** (1966) 120].

18) I. Affleck, M. Dine, Nucl. Phys. B**249** (1985) 361.

19) J. Madsen, *Phys. Rev. Lett.* **69** (1992) 571.

20) A.D. Dolgov, *Pisma Zh. Eksp. Teor. Fiz.* **29** (1979) 254.

21) A. D. Dolgov, S. H. Hansen, A. Yu. Smirnov, astro-ph/0503612.

22) L. Cucurull, J.A. Grifols, R. Toldra, *Astropart. Phys.* **4**, (1996) 391.

23) Eidelman S *et al*, 2004 *Phys. Lett.* **B592** 1

24) Fields B D and Olive K A, 2005 arXiv:astro-ph/9803297.

25) Izotov Y I and Thuan T X, 2004, *Astrophys. J.* **602**, 200

26) Trotta R and Hansen S H, 2004 *Phys. Rev. D* **69** 023509

27) Steigman G, astro-ph/0501591, To appear in the Proceedings of the ESO/Arcetri Workshop on "Chemical Abundances and Mixing in Stars in the Milky Way and its Satellites", eds., L. Pasquini and S. Randich (Springer-Verlag Series, "ESO Astrophysics Symposia").

28) A.D. Dolgov, F. Takahashi,*Nucl.Phys.* **B688** (2004) 189, hep-ph/0402066; A.D. Dolgov, F. Takahashi, Talk given at 12th International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 04), Tsukuba, Japan, 17-23 Jun 2004, hep-ph/0409299.

29) C. Lunardini and A.Yu. Smirnov, *Phys. Rev.* D**64** (2001) 073006.

30) V.M. Lobashev *et al*, *Phys. Lett.* B**460** (1999) 227; Ch. Weinheimer *et al*, *Phys. Lett.* B**460** (1999) 219.

31) W. C. Haxton, G. J. Stephenson Jr., *Prog. in Part. and Nucl. Phys.* **12** (1984) 409; M. Doi *et al.*, *Prog. of Theor. Phys. Suppl.* **83** (1985); J. D. Vergados *Phys. Rep.* **133** (1986) 1; F. Boehm and P. Vogel, *Physics of massive neutrinos*, Cambridge Univ. Press. 1987.

32) A. Barabash, A. Dolgov, P. Domin, F. Simkovic, A. Smirnov (in progress)